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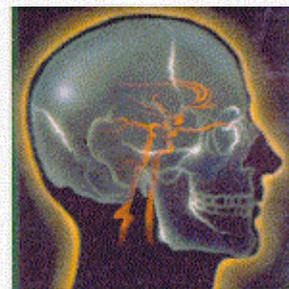
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Topological Dimension

U

Ultrametricity
Universality
Universal Computer
Universal Turing Machine
Unstable Equilibrium

Adaptation
Any change in the structure or function of an entity (say, a biological organism) that allows it to survive and reproduce more effectively in its environment.

Algorithmic Complexity
A measure of the complexity of a problem. Typically defined as the size of the smallest program that computes the given problem or that generates a complete description of it.

Animats

Artificial animals consisting of both software and hardware. Typically designed to be able to adapt to their environment over time.

Artificial Life

This is not a concept that is yet ready to be rigorously defined. The most concise, but still far from rigorous definition, is simply: life as synthesized by man rather than by nature. One of the basic tenets of this still-infant field is the belief that life is not unique to its biological (and, as yet, only known) form, but is a more general property of the organization of matter. Artificial life explores life as it could be as opposed to life as we know it to be.

Attractor

Dissipative dynamical systems are characterized by the presence of some sort of internal "friction" that tends to contract phase-space volume elements. Contraction in phase space allows such systems to approach a subset of the phase-space called an attractor as the elapsed time grows large. Attractors therefore describe the long-term behavior of a dynamical system. Steady state (or equilibrium) behavior corresponds to fixed-point attractors, in which all trajectories starting from the appropriate basin-of-attraction eventually converge onto a single point. For linear dissipative dynamical systems, fixed point attractors are the only possible type of attractor. Nonlinear systems, on the other hand, harbor a much richer spectrum of attractor types. For example, in addition to fixed-points, there may exist periodic attractors such as limit cycles. There is also an intriguing class of chaotic attractors called strange attractors that have a complicated geometric structure (see Chaos and Fractals).

Autonomous (or Adaptive-) Agent

An entity that, by sensing and acting upon its environment, tries to fulfill a set of goals in a complex, dynamic environment. Properties: (1) it can sense the environment through its sensors and act on the environment through its actuators; (2) it has an internal information processing and decision making capability; (3) it can anticipate future states and possibilities, based on internal models (which are often incomplete and/or incorrect); (4) this anticipatory ability often significantly alters the aggregate behavior of the system of which an agent is part. An agent's goals can take on diverse forms: (i) desired local states;(ii) desired end goals;(iii) selective rewards to be maximized; (iv) internal needs (or motivations) that need to be kept within desired bounds. Since a major component of an agent's environment consists of other agents, agents spend a great deal of their time adapting to the adaptation patterns of other agents.

Autoplectic Systems

Consider a dynamical system whose behavior appears random or chaotic. There are two ways in which an apparent randomness can occur: (1) external noise, so that if the evolution of the system is unstable, external perturbations amplify exponentially with time -such systems are called homoplectic; (2) internal mechanisms, so that the randomness is generated purely by the dynamics itself and does not depend on any external sources or require that randomness be present in the initial conditions -such systems are called autoplectic systems. An example of an autoplectic system is the one-dimensional, two-state, two neighbor Cellular Automaton rule-30, starting from a single non-zero site. The temporal sequence of binary values starting from that single non-zero initial seed are completely random, despite the fact that the evolution is strictly deterministic and the initial state is ordered.

Autopoiesis

Autopoiesis literally means "self-reproduction," and expresses a fundamental complementarity between structure and function. More precisely, the term refers to the dynamics of non-equilibrium structures; that is, organized states (sometimes also called dissipative structures) that remain stable for long periods of time despite matter and energy continually flowing through them. A vivid example of a nonequilibrium structure is the Great Red Spot on Jupiter, which is essentially a gigantic whirlpool of gases in Jupiter's upper atmosphere. This vortex has persisted for a much longer time (on the order of centuries) than the average amount of time any one gas molecule has spent within it.

Backpropagation Algorithm

The backpropagation algorithm is a learning rule for multi-layered Neural Networks, credited to Rumelhart and McClelland. The algorithm gives a prescription for adjusting the initially randomized set of synaptic weights (existing between all pairs of neurons in each successive layer of the network) so as to maximize the difference between the network's output of each input fact and the output with which the given input is known (or desired) to be associated. The backpropagation rule takes its name from the way in which the calculated error at the output layer is propagated backwards from the output layer to the Nth hidden layer to the (N1)th hidden layer, and so on. Because this learning process requires us to to "know" the correct pairing of input-output facts beforehand, this type of weight adjustment is called supervised learning.

Basin of Attraction

The basin of attraction is the ensemble of points P such that if the trajectory starts from P it approaches the Attractor.

Bifurcation

The splitting into two modes of behavior of a system that previously displayed only one mode. This splitting occurs as a control parameter is continuously varied. In the Logistic Equation, for example, a period-doubling bifurcation occurs whenever all the points of period-2n cycle simultaneously become unstable and the system becomes attracted to a new period-2n+1 cycle.

Boolean Function

A function that maps an n-tuple of binary values -(x1, x2, ..., xn), where xi = 0 or 1 for all i -to another binary value (either 0 or 1). There are clearly $2^{(2n)}$ possible Boolean functions that can defined for a given n-tuple.

Cantor Set

A simple example of a Fractal set of points having noninteger Hausdorff Dimension. For example, the triadic Cantor set is constructed as follows: take the unit interval (= [0,1]) and generate a new set by deleting the open interval (1/2, 2/3); that is, by deleting the middle third. Generate a new set by deleting the middle thirds (1/9, 2/9) and (7/9, 8/9) from the previous set with the middle third removed. The Cantor set is essentially what remains of the unit interval in the limit of generating successive

"middle third" deleted sets from the original set. It can be shown that the Fractal Dimension of this set is approximately equal to 0.6309.

Catastrophe Theory

Catastrophe theory, introduced by Thom in the 1960s, is a mathematical formalism for modeling nonlinear systems whose behavior is determined by the actions of a small number of driving parameters. In particular, it applies to systems that undergo either gradual or sudden changes in behavior due to gradually changing forces. It has been applied to many problems in mathematics, physics and the social sciences. Thom called the sudden changes that take place in a system "catastrophes" and developed a theory as a method of analyzing and classifying these changes. Thom's theorem asserts that the stationary state behavior of all systems that have up to four control parameters (or input variables) and two behavior (or output) variables, and which also have an associated potential function, can be described using one of seven elementary catastrophes.

Cellular Automata

Cellular automata (CA) are a class of spatially and temporally discrete, deterministic mathematical systems characterized by local interaction and an inherently parallel form of evolution. First introduced by von Neumann in the early 1950s to act as simple models of biological selfreproduction, CA are prototypical models for complex systems and processes consisting of a large number of identical, simple, locally interacting components. The study of these systems has generated great interest over the years because of their ability to generate a rich spectrum of very complex patterns of behavior out of sets of relatively simple underlying rules. Moreover, they appear to capture many essential features of complex self-organizing cooperative behavior observed in real systems. Although much of the theoretical work with CA has been confined to mathematics and computer science, there have been numerous applications to physics, biology, chemistry, biochemistry, and geology, among other disciplines. Some specific examples of phenomena that have been modeled by CA include fluid and chemical turbulence, plant growth and the dendritic growth of crystals, ecological theory, DNA evolution, the propagation of infectious diseases, urban social dynamics, forest fires, and patterns of electrical activity in neural networks. CA have also been used as discrete versions of partial differential equations in one or more spatial variables.

Cellular Games

A cellular game is a dynamical system in which sites of a discrete lattice play a "game" with neighboring sites. Strategies may be deterministic or stochastic. Success is usually judged according to a universal and fixed criterion. Successful strategies persist and spread throughout the lattice; unsuccessful strategies disappear.

Chaos

Deterministic chaos refers to irregular or chaotic motion that is generated by nonlinear systems evolving according to dynamical laws that uniquely determine the state of the system at all times from a knowledge of the system's previous history. It is important to point out that the chaotic behavior is due neither to external sources of noise nor to an infinite number of degrees-of-freedom nor to quantum-mechanical-like uncertainty. Instead, the source of irregularity is the exponential divergence of initially close trajectories in a bounded region of . This sensitivity to initial conditions is sometimes popularly referred to as the "butterfly effect," alluding to the idea that chaotic weather patterns can be altered by a butterfly flapping its wings. A practical implication of chaos is that its presence makes it essentially impossible to make any longterm predictions about the behavior of a dynamical system: while one can in practice only fix the initial conditions of a system to a finite accuracy, their errors increase exponentially fast.

Chaotic Control

It has recently been suggested that the extreme sensitivity of chaotic systems to small perturbations to to initial conditions (the so called "butterfly effect") can be exploited to stabilize regular dynamic behaviors and to effective "direct" chaotic trajectories to a desired state. This is a capability that has no counterpart in nonchaotic systems for the ironic reason that the trajectories in nonchaotic systems are stable and thus relatively impervious to desired control. A recent survey article (Grebogi, Ott, et. al.) lists applications for communications (in which chaotic fluctuations can be put to use to send controlled, pre-planned signals), for physiology (controlling chaos in heart rhythms), for fluid mechanics and chemical reactions. As another recent example, a few years ago NASA used small amounts of residual hydrazine fuel to steer the ISEE3/ICE spacecraft to its rendezvous with a comet 50 million miles away. This was possible because of the sensitivity of the three-body problem of celestial mechanics to small perturbations.

Classifier Systems

Classifier systems were introduced by John Holland as an attempt to apply Genetic Algorithms to cognitive tasks. They are similar to production systems of the "if...then" variety in artificial intelligence. A classifier system typically consists of (1) a set of detectors (or input devices) that provide information to the system about the state of the external environment, (2) a set of effectors (or output devices) that transmit the classifier's conclusions to the external environment, (3) a set of rules (or classifiers), consisting of a condition and action, and (4) a list of messages. Learning is supervised as in multilayered Neural Networks.

Class-P Problems

The Computational Complexity of a problem is defined as the time it takes for the fastest program running on a universal computer -as measured in number of computing steps, say N -to compute the solution to the problem. The complexity is then classified according to how fast N grows as a function of the problem size, s. The first non-trivial class of problems -class-P - consists of problems for which the computation time increases as some polynomial function of s. Problems that can be solved with polynomial-time algorithms are called tractable; if they are solvable but are not in the class-P, they are called intractable.

Co-Adaptation/Co-Evolution

The evolutionary process of a biological species in nature is often described as though that species were trying to adapt to a fixed environment. However, such a description only crudely approximates what really happens. In nature, the "environment" consists of both a relatively (but not completely) stable physical environment as well as other species of organisms that are simultaneously trying to adapt to their environment. The actions of each of these other species typically affects the actions of

all other species that occupy the same physical environment. In biology (and hence Artificial Life and studies involving Genetic Algorithms), the terms "coadaptation" and "co-evolution" are sometimes used to refer to the fact that all species simultaneously co-adapt and co-evolve in a given physical environment.

Complex Adaptive Systems

Macroscopic collections of simple (and typically nonlinearly) interacting units that are endowed with the ability to evolve and adapt to a changing environment.

Complexity

An extremely difficult "I know it when I see it" concept to define, largely because it requires a quantification of what is more of a qualitative measure. Intuitively, complexity is usually greatest in systems whose components are arranged in some intricate difficult-to-understand pattern or, in the case of a dynamical system, when the outcome of some process is difficult to predict from its initial state. In its lowest precisely when a system is either highly regular, with many redundant and/or repeating patterns or when a system is completely disordered. While over 30 measures of complexity have been proposed in the research literature, they all fall into two general classes: (1) Static Complexity -which addresses the question of how an object or system is put together (i.e. only purely structural informational aspects of an object), and is independent of the processes by which information is encoded and decoded; (2) Dynamic Complexity -which addresses the question of how much dynamical or computational effort is required to describe the information content of an object or state of a system. Note that while a system's static complexity certainly influences its dynamical complexity, the two measures are not equivalent. A system may be structurally rather simple (i.e. have a low static complexity), but have a complex dynamical behavior.

Computational Complexity

Computational complexity measures the time and memory resources that a computer requires in order to solve a problem. A somewhat more robust measure may be defined by invoking the Universal Turing Machine. The Computational Complexity of a problem is then defined as the time it takes for the fastest program running on a universal computer (as measured in number of computing steps) to compute the solution to the problem.

Computational Irreducibility

Much of theoretical physics has traditionally been concerned with trying to find "shortcuts" to nature. That is to say, with trying to find methods that are able to reproduce a final state of a system by knowing the initial state but without having to meticulously trace out each step from the initial to final states. The fact that we can write down a simple parabola as a path a thrown object makes in a gravitational field is an example of an instance where this might be possible. Clearly such shortcuts ought to be possible in principle if the calculation is more sophisticated than the computations the physical system itself is able to make. But consider a computer. Because a computer is itself physical system, it can determine the outcome of its evolution only by explicitly following it through. No shortcut is possible. Such computational irreducibility occurs whenever a physical system can act as a computer. In such cases, no general predictive ability is possible. Computational irreducibility implies that there is a highest level at which abstract models of physical systems can be made. Above that level, one can model only by explicit simulation.

Computational Universality

Computational universality is a property of a certain class of computers such that changes in input alone allow any computable function to be evaluated without any change in internal construction. Universal computers can thus simulate the operation of any other computer, given that their input is suitably coded. Conway's Life Game, for example, has been shown to be a universal computer. This means that with a proper selection of initial conditions (i.e. the initial distribution of "live" and "dead" cells), Life can be turned into a general purpose computer. This fact fundamentally limits the overall predictability of Life's behavior. The Halting Theorem, for example, asserts that there cannot exist a general algorithm for predicting when a computer will halt its execution of a given program. Given that Life is a universal computer -so that the Halting theorem applies -this means that one cannot, in general, predict whether a particular starting configuration of live and dead cells will eventually die out. No shortcuts are possible, even in principle.

Conservative Dynamical Systems

In contrast to Dissipative Dynamical Systems, conservative systems preserve Phase Space volumes and hence cannot display any attracting regions in phase space; there can be no fixed points, no limit cycles and no strange attractors. There can nonetheless be chaotic motion in the sense that points along particular trajectories may show sensitivity to initial conditions. A familiar example of a conservative system from classical mechanics is that of a Hamiltonian system.

Correlation Dimension

Cost Function

In optimization problems, the cost function measures how good a particular solution to the problem is; the higher its value the better the solution. Also called the fitness function.

Coupled-Map Lattices

Generic Cellular Automata (CA) are dynamical systems in which space, time and the local state space are all discretized. Coupled-map lattices are simple generalizations of CA in which space and time remain discrete, but in which the individual site values are allowed to take on continuous values.

Criticality

"Criticality" is a concept borrowed from thermodynamics. Thermodynamic systems generally get more ordered as the temperature is lowered, with more and more structure emerging as cohesion wins over thermal motion. Thermodynamic systems can exist in a variety of phases -gas, liquid, solid, crystal, plasma, etc. -and are said to be critical if poised at a phase transition. Many phase transitions have a critical point associated with them, that separates one or more phases. As a thermodynamic system approaches a critical point, large structural fluctuations appear despite the fact the system is driven

only by local interactions. The disappearance of a characteristic length scale in a system at its critical point, induced by these structural fluctuations, is a characteristic feature of thermodynamic critical phenomena and is universal in the sense that it is independent of the details of the system's dynamics. (See Self-Organized Criticality)

Crossover Operator

One of three basic genetic operations used in Genetic Algorithms. Reproduction makes a set of identical copies of a given chromosome, where the number of copies depends on the chromosome's fitness. The crossover operator exchanges subparts of two chromosomes, where the position of the crossover is randomly selected, and is thus a crude facsimile of biological sexual recombination between two single-chromosome organisms. The mutation operator randomly flips one or more bits in the chromosome, where the bit positions are randomly chosen.

Dissipative Structure

An organized state of a physical system whose integrity is maintained while the system is far from equilibrium. *Example: the great Red-Spot on Jupiter*.

Dissipative Dynamical Systems

Dissipative systems are dynamical systems that are characterized by some sort of "internal friction" that tends to contract phase space volume elements. Phase space contraction, in turn, allows such systems to approach a subset of the space called an Attractor (consisting of a fixed point, a periodic cycle, or Strange Attractor), as time goes to infinity.

Edge-of-Chaos

The phrase "edge-of-chaos" refers to the idea that many complex adaptive systems, including life itself, seem to naturally evolve towards a regime that is delicately poised between order and chaos. More precisely, it has been used as a metaphor to suggest a fundamental equivalence between the dynamics of phase transitions and the dynamics of information processing. Water, for example, exists in three phases: solid, liquid and gas. Phase transitions denote the boundaries between one phase and another. Universal computation -that is, the ability to perform general purpose computations and which is arguably an integral property of life exists between order and chaos. If the behavior of a system is too ordered, there is not enough variability or novelty to carry on an interesting calculation; if, on the other hand, the behavior of a system is too disordered, there is too much noise to sustain any calculation. Similarly, in the context of evolving natural ecologies, "edge-of-chaos" refers to how -in order to successfully adapt -evolving species should be neither too methodical nor too whimsical or carefree in their adaptive behaviors. The best exploratory strategy of an evolutionary "space" appears at a phase transition between order and disorder. Despite the intuitive appeal of the basic metaphor, note that there is currently some controversy over the veracity of this idea.

Emergence

Emergence refers to the appearance of higher-level properties and behaviors of a system that while obviously originating from the collective dynamics of that system's components -are neither to be found in nor are directly deducable from the lower-level properties of that system. Emergent properties are properties of the "whole" that are not possessed by any of the individual parts making up that whole. Individual line of computer code, for example, cannot calculate a spreadsheet; an air molecule is not a tornado; and a neuron is not conscious. Emergent behaviors are typically novel and unanticipated.

Entropy

A measure of the degree of randomness or disorder in a system. Determines a system's capacity to evolve irreversibly in time. Specific definitions vary depending on the type of system considered. Examples: (1) in statistical systems, the entropy is proportional to the logarithm of the total number of possible states with the same energy as the state under consideration.; (2) in classical thermodynamics, the differential change in entropy of a system near equilibrium is the differential change in absorbed heat divided by the system temperature; (3) in nonlinear deterministic dynamical systems, the Kolmogorov-Sanai entropy is often used as a measure. It is defined as the sum of the positive Lyapunov Exponents of the system.

Ergodic System

An ergodic dynamical system is one whose trajectory eventually "covers" the entire phase space. Put another way, given any point P in the phase space, the trajectory will approach P arbitrarily closely for sufficiently large times t.

Ergodic Theory

A branch of mathematics that uses statistical concepts to describe average properties of deterministic dynamical systems.

Evolution

A general term referring to the dynamical unfolding of behavior over time. Darwinian evolution refers to the unfolding of higher (i.e. more complex) life forms out of lower life forms.

Evolutionary Programming

Evolutionary programming is an early variant of genetic algorithms and is mainly distinguished from the conventional genetic algorithm by not incorporating crossover as an operator.

Evolutionary Stable Strategy

A concept from a generalized form of Game Theory. Animals are endowed with a finite set of possible strategies that they can use in their interactions with other animals. Strategies may be "pure," in which the animal acts according to a prescribed set of instructions in all contexts, or "mixed," in which the animal adopts different strategies with different probabilities. The evolutionary stable strategy (ESS) is a strategy, or set of strategies such that if it is adopted by all animals no other strategy can invade the population.

Finite Automata

Human languages can, conceptually, be regarded as a set of rules for constructing sequences of symbols according to a fixed set of rules of composition in order to convey meaning. One can therefore consider using a Cellular Automaton as a formalism for studying the abstract properties of language. To be more precise, a finite automaton M is defined to consist of a finite alphabet A, a finite set of states X, and a state-transition function f: X x A -X that gives the next state given the current state and the current input symbol. (There is also a set T in X, which is the set of final or accepting states of the automaton.)

Fitness Landscape

A name for the landscape representing the fitness measure (or Cost Function) of a problem. Examples: Traveling Salesman Problem, survivability of a real or virtual creature.

Flicker(or 1/f-) Noise

Whenever the power spectral density, S(f), scales as $f^{(-1)}$, the system is said to exhibit 1/f-noise (or flicker-noise). Despite being found almost everywhere in nature -1/f-noise has been observed in the current fluctuations in a resistor, in highway traffic patterns, in the price fluctuations on the stock exchange, in fluctuations in the water level of rivers, to name just a few instances -there is currently no fundamental theory that adequately explains why this same kind of noise should appear in so many diverse kinds of systems. What is clear is that since the underlying dynamical processes of these systems are so different, the common bond cannot be dynamical in nature, but can only be a kind of "logical dynamics" describing how a system's degrees-offreedom all interact. Self-Organized Criticality may be a fundamental link between temporal scale invariant phenomena and phenomena exhibiting a spatial scale invariance. Bak, et. al., argue that 1/f noise is actually not noise at all, but is instead a manifestation of the intrinsic dynamics of Self-Organized Critical systems.

Fractals

Fractals are geometric objects characterized by some form of self-similarity; that is, parts of a fractal, when magnified to an appropriate scale, appear similar to the whole. Coastlines of islands and continents and terrain features are approximate fractals. The Strange Attractors of nonlinear dynamical systems that exhibit deterministic Chaos typically are fractals.

Fractal Dimension

Suppose a set can be covered by a finite number N of segments of length L. There is a simple scaling relationship between these two numbers. For a line segment, L grows as 1/R; for a square, L grows as $1/R^2$; for a cube, L grows as $1/R^3$, and so on. The fractal dimension D is defined by generalizing this intuitive scaling: $D = \lim_{R\to 0} \ln(N)/(\ln(1/R))$, where ln(x) is the natural logarithm. Sometimes also called the Hausdorff dimension or the Kolmogorov capacity.

Frustration

In Spin Glasses, a phenomenon in which individual magnetic moments receive competing ordering instructions via different routes, because of the variation of the interaction between pairs of atomic moments with separation.

Fuzzy Logic

Fuzzy set theory provides a formalism in which the conventional binary logic based on choices "yes" and "no" is replaced with a continuum of possibilities that effectively embody the alternative "maybe". Formally, the characteristic function of set X defined by f(x) =1 for all x in X and f(x)=0 for all x not in X is replaced by the membership function 0

Genetic Algorithms

Genetic algorithms are a class of heuristic search methods and computational models of adaptation and evolution based on natural selection. In nature, the search for beneficial adaptations to a continually changing environment (i.e. evolution) is fostered by the cumulative evolutionary knowledge that each species possesses of its forebears. This knowledge, which is encoded in the chromosomes of each member of a species, is passed from one generation to the next by a mating process in which the chromosomes of "parents" produce "offspring" chromosomes. Genetic algorithms mimic and exploit the genetic dynamics underlying natural evolution to search for optimal solutions of general combinatorial optimization problems. They have been applied to the travelling salesman problem, VLSI circuit layout, gas pipeline control, the parametric design of aircraft, neural net architecture, models of international security, and strategy formulation.

Genetic Programming

Genetic programming is essentially an application of genetic algorithms to computer programs. Typically the genome is represented by a LISP expression, so that what evolves is a population of programs, rather than bit-strings as in the case of a usual genetic algorithm.

Genotype

The genetic instruction code of an individual.

Hausdorff Dimension

For an operational definition of Hausdorff dimension, proceed as follows: Suppose a set can be covered by a finite number N of segments of length L. There is a simple scaling relationship between these two numbers. For a line segment, L grows as 1/R; for a square, L grows as $1/R^2$; for a cube, L grows as $1/R^3$, and so on. The Hausdorff dimension D is defined by generalizing this intuitive scaling: $D = \lim_{R\to 0} \ln(N)/(\ln(1/R))$, where ln(x) is the natural logarithm. Sometimes also called the fractal dimension or the Kolmogorov capacity.

Hierarchy

Hierarchies consist of levels each of which include all lower levels; i.e. systems within systems within systems…within the total system in question. Evolution in complex systems leads to differentiation in multilevel hierarchic systems.
Holarchy

Homoclinic Point

A point in Phase Space of a nonlinear dynamical system that evolves to a point of unstable equilibrium in infinite time.

Homoclinic Orbit

The ensemble of points in the [Phase Space](#) of a nonlinear dynamical system that all evolve to a point of unstable equilibrium after an infinite time.

Hopf-Bifurcation

In the [Logistic Map](#), a fixed point may lose its stability by splitting (or bifurcating) into a pair of points that form a period two orbit. Another common way in which a point may become unstable is by effectively turning into a small circle that then increases in size, deforms and becomes unstable as the controlling parameter is increased. This is called the Hopf Bifurcation.

Hypercycle

A scenario for the origin of self-replicating molecular systems proposed by Manfred Eigen. The scenario involves template-instructed replicating cycles consisting of feedback loops in which molecule A generates molecule B, molecule B generates molecule C, and molecule C generates molecule A, and so on.

Information Dimension

Partition a d-dimensional [Phase Space](#) into boxes of volume ed. The probability of finding a point of an [Attractor](#) in box i is $p_i(e)$ = $N_i(e)/N(e)$, where $N_i(e)$ is the number of points in the ith box and $N(e)$ is the total number of non-empty boxes. $p_i(e)$ is the relative frequency with which the ith box is visited. The amount of information required to specify the state of the system to within an accuracy "e" (or, equivalently, the information gain in making a measurement that is uncertain by an amount "e"), is given by . The information dimension, D_I, of an attractor is then defined to be D_I = lim e--0 I(e)/ln(1/e).

Information Theory

Like the physically primitive notions of mass and energy of a particle, the information content, I, of an arbitrary measurement or message composed of particular symbol sequence, is itself a primitive concept. While the roots of information theory extend back to the definition of the classical entropy of a physical system introduced by Clausius in 1864 and Boltzman's probabilistic re-interpretation of classical entropy in 1896, the mathematical formalism for measuring I is due largely to a seminal 1948 paper by Claude E. Shannon. Within the context of sending and receiving messages in a communication system, Shannon was interested in finding a measure of the information content of a received message. Shannon's approach was to obtain a measure of the reduction of uncertainty given some a-priori knowledge of the symbols being sent. Suppose we are given N different and a-priori equally likely possible outcomes. A measure of the information gain, I, is obtained by required that I be additive for independent events. That is to say, if there are two independent sets of outcomes N1 nd N2, so that the total number of outcomes is N = N1 + N2, it is required that I(N1 * N2) = I(N1) + I(N2). This requirement is uniquely satisfied by the function I= c log(N), where "c" is an arbitrary constant.

Intermittency

A term used in the study of nonlinear dynamical systems describing the changes between quiet, regular periods of activity (called the laminar phase) and periods of wild, chaotic oscillation (called bursts). Intermittency is a common route to [chaos](#) in physical systems.

Kolmogorov Entropy

The Kolmogorov entropy (or K-entropy) is a useful measure by which to characterize chaotic motion in an arbitrary-dimensional [Phase Space](#). Loosely speaking, the K-entropy is proportional to the rate at which information about the state of a dynamical system is lost in the course of time. It is related to the average [Lyapunov Exponent](#)Lyapunov Exponent, which measures the exponential rate of divergence of nearby trajectories.

Lattice Gas Models

Lattice gases are micro-level rule-based simulations of macro-level fluid behavior. The [Navier-Stokes Equations](#), the fundamental equations describing incompressible fluid flow, are in general analytically intractable. Lattice-gas models provide a powerful new tool in modeling real fluid behavior. The idea is to reproduce the desired macroscopic behavior of a fluid by modeling the underlying microscopic dynamics. In order to achieve an Emergence of a suitable macrodynamics out of a discrete microscopic substrate, one must have three basic ingredients: (1) local thermodynamic equilibrium, (2) conservation laws, and (3) a "scale separation" between the levels at which the microscopic dynamics takes place (among kinetic variables living on a micro-lattice) and the collective motion itself appears (defined by hydrodynamical variable on a macro-lattice). Another critical feature is the symmetry of the underlying lattice. While there are many basic variants of the model, one can show that there is a well-defined minimal set of rules that define a lattice-gas system whose macroscopic behavior reproduces that predicted by the Navier-Stokes equations exactly.

Life Game

Invented by the mathematician John Conway, Life is arguably the most widely known [Cellular Automaton](#). It was extensively popularized by Martin Gardner in his "Mathematical Games" department in Scientific American in the early 1970s. Life is "played" using the eight nearest-neighbors on a lattice, and consists of (1) seeding the lattice with some pattern of "live" and "dead" cells, and (2) simultaneously (and repeatedly) applying the following three rules to each cell of the lattice at discrete time steps: (i) Birth: replace a previously dead cell with a live one if exactly 3 of its neighbors are alive; (ii) Death: replace a previously live cell with a dead one if either (1) the living cell has no more than one live neighbor (i.e. it dies of isolation), or (2) the living cell has more than three neighbors (i.e. it dies of overcrowding); (iii) Survival: retain living cells if they have either 2 or 3 neighbors One of the most intriguing patterns in Life is an oscillatory propagating pattern known as the "glider." It consists of 5 "live" cells and reproduces itself in a diagonally displaced position once every four iterations. When the states of Life are projected onto a screen in quick succession by a fast computer, the glider gives the appearance of "walking" across the screen. The propagation of this pseudo-stable structure can also be seen as a self-organized emergent property of the system.

Limit-Cycle

An Attractor describing regular (i.e. periodic or quasi-periodic) temporal behavior.

Lindenmeyer (or L-) Systems

L-systems were introduced by Aristid Lindenmeyer in 1968 as a model for the cellular development of filamentous plants. In simplest terms, L-systems consist of production rules for rewriting abstract strings of symbols. They can be thought of as generalized Cellular Automata in which the number of sites can increase over time.

Logistic Equation

The logistic map is one of the simplest (continuous and differentiable) nonlinear systems that captures most of the key mechanisms responsible for producing deterministic chaos. It is a one-dimensional nonlinear discrete difference equation with a single control parameter, a: $x_{n+1} = a x_n (1-x_n)$, where $0 < x_0 < 1$ and $0 < a < 4$. The logistic equation undergoes a sequence of period-doublingBifurcations followed by regions of deterministic chaos as a is varied between the values 0 and 4. Some aspects of this behavior such as the ratio of bifurcation intervals as chaos is approached are Universal; that is, are independent of the details of the system.

Lotka-Volterra Equations

In 1926, Volterra proposed a simple model for the predation of one species by another to explain the oscillatory level of certain fish in the Atlantic. If N(t) is the prey population and P(t) is the predator population at time t then Volterras's model is $dN/dt = N(a - bP)$, $dP/dt = P(cN - d)$, where a,b,c, and d are positive constants. The model assumes: (1) prey in absence of predation grows linearly with N (i.e. in Malthusian fashion); (2) predation reduces prey's growth rate by a term proportional to the prey and predation populations; (3) the predator's death rate, in the absence of prey, decays exponentially; (4) the prey's contribution to the predator's growth rate is proportional to the available prey as well as to the size of the predator population. The system of equations is known as the Lotka-Volterra equations because Lotka derived the same equations in 1920 for a chemical reaction he believed to exhibit periodic behavior.

Lyapunov Exponent

A fundamental property of chaotic dynamics is sensitivity to small changes to initial conditions. Initially closely separated starting conditions evolving along regular dynamical trajectories diverge only linearly in time; a chaotic evolution, on the other hand, leads to exponential divergence in time. Lyapunov exponents quantify this divergence by measuring the mean rate of exponential divergence of initially neighboring trajectories. A trajectory of a system with a negative Lyapunov exponent is stable and will converge to an Attractor exponentially with time. The magnitude of the Lyapunov exponent determines how fast the attractor is approached. A trajectory of a system with a positive Lyapunov exponent is unstable and will not converge to an attractor. The magnitude of the positive Lyapunov exponent determines the rate of exponential divergence of the trajectory.

Markov Process

A Markov process is a process for which, if the present is given, the future and past are independent of each other. More precisely, if $t_1 < ... < t_n$ are parameter values, and if $1 < j < n$, then the sets of random variables $[x(t_1), ..., x(t_{j-1})]$ and $[x(t_{j+1}), ..., x(t_n)]$ are mutually independent for given $x(t_j)$. Equivalently, the conditional probability distribution of $x(t_n)$ for given $x(t_1), ..., x(t_{n-1})$ depends only on the specified value of $x(t_{n-1})$ and is in fact the conditional probability distribution of $x(t_n)$, given $x(t_{n-1})$. An important and simple example is the Markov chain, in which the number of states is finite or denumerably infinite.

Maximum Entropy

The principle of maximum entropy states that when one has only partial information about the probabilities of possible outcomes of an experiment, one should choose the probabilities so as to maximize the uncertainty about the missing information. Put another way, since entropy is a measure of randomness, one should choose the most random distribution subject to whatever constraints are imposed on the problem.

Mean-Field Theory

In a mean field approximation a system is assumed to be determined by the average properties of the system as a whole. In a mean-field-theoretic description of a thermodynamic system, for example, all particles are considered to contribute equally to the potential at each site. Therefore, the mean field theory essentially assumes the intermolecular interaction to be of infinite range at all temperatures. The mean field theories are qualitatively quite successful in that they predict the existence of critical points and power law dependence of the various thermodynamic quantities near the critical point. They generally become more quantitatively successful as the dimensionality of the system increases.

Multifractal

The simplest fractal sets are characterized by some form of self-similarity, in which parts, when magnified by a constant r, appear similar to the original whole. The more general class of fractals are really multi-scale fractals, or multifractals, which are characterized by multiple subdivisions of the original into N objects, each magnified by by a different factor r_i, i=1,2,...,N.

Navier-Stokes Equations

These are a set of analytically intractable coupled nonlinear partial differential equations describing fluid flow.

Neural Networks

Neural nets represent a radical new approach to computational problem solving. The methodology they represent can be contrasted with the traditional approach to artificial intelligence (AI). Whereas the origins of AI lay in applying conventional serial processing techniques to high-level cognitive processing like concept-formation, semantics, symbolic processing, etc. -or in a top-down approach -neural nets are designed to take the opposite -or bottom-up -approach. The idea is to have a human-like reasoning emerge on the macro-scale. The approach itself is inspired by such basic skills of the human brain as its ability to continue functioning with noisy and/or incomplete information, its robustness or fault tolerance, its adaptability to changing environments by learning, etc. Neural nets attempt to mimic and exploit the parallel processing capability of the human brain in order to deal with precisely the kinds of problems that the human brain itself is well adapted for.

Nonlinearity

If f is a nonlinear function or an operator, and x is a system input (either a function or variable), then the effect of adding two inputs, x1 and x2, first and then operating on their sum is, in general, not equivalent to operating on two inputs separately and then adding the outputs together; i.e. . Popular form: the whole is not necessarily equal to the sum of its parts. Dissipative nonlinear dynamic systems are capable of exhibiting self-organization and chaos.

NP-Hard Problems

A class of problems, known as nondeterministic polynomial time -or class-NP -problems, that may not necessarily be solvable in polynomial time, but the actual solutions to which may be tested for correctness in polynomial time.

NP-Complete

Just as there are universal computers that, given a particular input, can simulate any other computer, there are NP-complete problems that, with the appropriate input, are effectively equivalent to any NP-hard problem of a given size. For example, Boolean "satisfiability" -i.e. the problem of determining truth values of the variable's of a Boolean expression so that the expression is true -is known to be an NP-complete problem.

Order Parameter

An order parameter is a scalar or vector parameter associated with a continuous phase transition that determines the physical nature of the transition. It has the value zero in the random state (typically above the transition temperature) and takes on a nonzero value in the ordered state (typically below the transition). In the case of a fluid, for example, the order parameter is a scalar and is the difference in density between the liquid and vapor phases.

Percolation Theory

Percolation Theory represents one of the simplest models of a disordered system. Consider a square lattice, where each site is occupied randomly with probability p or empty with probability 1-p. Occupied and empty sites may stand for very different physical properties. For simplicity, let us assume that the occupied sites are electrical conductors, the empty sites represent insulators, and that electrical current can flow between nearest neighbor conductor sites. At low concentration p, the conductor sites are either isolated or form small clusters of nearest neighbor sites. Two conductor sites belong to the same cluster if they are connected by a path of nearest neighbor conductor sites, and a current can flow between them. At low p values, the mixture is an insulator, since a conducting path connecting opposite edges of the lattice does not exist. At large p values, on the other hand, many conduction paths between opposite edges exist, where electrical current can flow, and the mixture is a conductor. At some concentration in between, therefore, a threshold concentration pc must exist where for the first time electrical current can percolate from one edge to the other. Below pc, we have an insulator; above pc we have a conductor. The threshold concentration is called the percolation threshold, or, since it separates two different phases, the critical concentration.

Petri Nets

Petri nets are abstract models used to represent parallel systems and processes. They are typically described using directed graphs (i.e. graphs whose edges are depicted by arrows showing a direction of information flow). More precisely, a petri net is a seven-tuple (P, T, V, f, g, N, m), where (1) P is a nonempty finite set of nodes, (2) T is a nonempty finite set of transitions, (3) V is a valuation space {0,1}, (4) f is a binary function used in determining the connections from nodes to transitions (i.e. f: P x T -V, and if f(p,t)=1 then node P connects to transition T, otherwise not), (5) g is a binary function used in determining the transitions to connect to nodes (i.e. g: T x P -V and a connection is made from t to p if and only if g(t,p)=1), (6) N is a set of markings {0,1,2,...}, and (7) m is the initial marking function, m: P -N.

Phase Space

A mathematical space spanned by the dependent variables of a given dynamical system. If the system is described by an ordinary differential flow the entire phase history is given by a smooth curve in phase space. Each point on this curve represents a particular state of the system at a particular time. For closed systems, no such curve can cross itself. If a phase history a given system returns to its initial condition in phase space, then the system is periodic and it will cycle through this closed curve for all time. Example: a mechanical oscillator moving in one-dimension has a two-dimensional phase space spanned by the position and momentum variables.

Phase Transition

An abrupt change in a system's behavior. A common example is the gas-liquid phase transition undergone by water. In such a transition, a plot of density versus temperature shows a distinct discontinuity at the critical temperature marking the transition point. Similar behavior can be seen in systems described by ordinary differential flows and discrete mappings. In nonlinear dynamical systems, the transition from self-organizing to chaotic behavior is sometimes referred to as a phase transition (or, more specifically, as an order-disorder transition).

Phenotype

The overall attributes of an organism arising from the interaction of its Genotype with the environment.

Poincare Map

A dynamical system is usually defined as a continuous flow, that is (1) is completely defined at all times by the values of N variables -x1(t), x2(t), ..., xN(t), where xi(t) represents any physical quantity of interest, and (2) its temporal evolution is specified by an autonomous system of N, possibly coupled, ordinary first-order differential equations. Once the initial state is specified, all future states are uniquely defined for all times t. A convenient method for visualizing continuous trajectories is to construct an equivalent discrete-time mapping by a periodic "stroboscopic" sampling of points along a trajectory. One way of accomplishing this is by the so-called Poincare map (or surface-of-section) method. Suppose the trajectories of the system are curves that live in a three-dimensional Phase Space.The method consists essentially of keeping track only of the intersections of this curve with a two-dimensional plane placed somewhere within the phase space.

Prisoner's Dilemma

The prisoner's dilemma is a two person non-zerosum game that has been widely used in experimental and theoretical investigations of
cooperative behavior. Two persons suspected of a crime are caught, but there is not enough evidence to sentence them unless one of them confesses. If they are both quiet (or cooperate, C), both will have to be released. If one confesses (defects, D) but the other does not, the one who confesses will be released but the other will be imprisoned for a long time. Finally, if both confess, both will be imprisoned, but for a shorter time. It is assumed that the prisoner's make their respective choices separately and independently of one another. If the game is "played" once, each player find defection to be the optimal behavior, regardless of what his opponent chooses to do. Finding the optimal strategy to follow over time, however, is considerably more difficult.

Probabilistic CA

Cellular Automata for which the deterministic state transitions are replaced with specifications of the probabilities of cell-value assignments. For such systems, the focus of analysis shifts from studying evolutions of arbitrary initial states to studying ensembles of trajectories.

Punctuated Equilibrium

A theory introduced in 1972 to account for what the fossil record appears to suggest are a series of irregularly spaced periods of chaotic and rapid evolutionary change in what are otherwise long periods of evolutionary stasis. Some Artificial Life studies suggest that this kind of behavior may be generic for evolutionary processes in complex adaptive systems.

Quasiperiodic

Characterizes behavior of a dynamical system that is almost, but not quite, periodic.Quasiperiodic regions of Phase Space are frequently linked together to form a Strange Attractor. The transition between such quasiperiodic regions is characterized by the crossing of a Homoclinic Point. Quasiperiodicity often results when nonlinear dynamical systems are driven by periodic driving forces with periods that are incommensurate with (i.e. not a rational fraction of) the system response time.

Random Boolean Networks

A size N random Boolean network of size k generalizes the basic binary Cellular Automata model by evolving each site variable $x_i = 0$ or 1 according to a randomly selected Boolean Function of k inputs. Since there are two choices for every combination of states of the k inputs at each site, the Boolean function is randomly selected from among the $2^{(2n)}$ possible Boolean functions of k inputs. This model was first introduced by Kauffman in 1969 in a study of cellular differentiation in a biological system (binary sites were interpreted as elements of an ensemble of genes switching on and off according to some set of random rules). Since its conception, however, related models have found wide application in an increasingly large domain of diverse problems. Such models of strongly disordered systems exhibit remarkable and unexpected order.

Reaction-Diffusion Models

Reaction-diffusion systems, the first studies of which date back to the 1950s, often exhibit a variety of interesting spatial patterns that evolve in self-organized fashion. One of the most famous reaction-diffusion systems -widely regarded as the prototypical example of oscillating chemical reactions -is the so-called Belousov-Zhabotinskii (or BZ) reaction. The BZ model involves the reaction of bromate ions with an organic substrate (typically malonic acid) in a sulfuric acid solution with cerium (or some other metal-ion catalyst). When this mixture is allowed to react exothermally at room temperature, interesting spatial and temporal oscillations (i.e. chemical waves) result. The system oscillates, changing from yellow to colorless to back to yellow about twice a minute with the oscillations typically lasting for over an hour (until the organic substrate is exhausted). A number of Cellular Automata models have been found that exhibit BZ-like spatial waves.

Relativistic Information Theory

Relativistic information theory is a concept introduced by Jumarie and has been suggested as a possible formalism for describing certain aspects of military command and control processes by Woodcock and Dockery. The basic idea is that a generalized entropy is endowed with four components, so that it is equivalent to a four-vector and may be transformed by a Lorentz transformation (As in relativity). These four components consists of: (1) the external entropy of the environment (Ho), (2) the internal entropy of the system (Hi), (3) system goals, and (4) the internal transformation potential, which measures the efficiency of the system's internal information transformation. An additional factor, called the organizability, plays the role of "velocity." Woodcock and Dockery show that it is possible to use relativistic information theory to compare the relative command and control system response of two command structures to the world around them. The quantity of interest is dHi/dHo, or the rate of change of the internal information environment with respect to changes in the surrounding environment.

Scaling Laws

Theoretical studies of critical phenomena have focused on predicting the value of critical exponents. One of the most important ideas is the scaling hypothesis. This hypothesis is model-independent and applicable to all critical systems. The underlying assumption is that the long-range correlation of the order parameter, such as the density fluctuation in a fluid system near the critical temperature, is responsible for all singular behavior. This assumption leads to a particular functional form for the equation of state near the critical point.

Search Space

The variation of the Cost Function can be imagined to be a landscape of potential solutions to a problem where the height of each feature represents its cost. This landscape is sometimes referred to as the search space.

Self-Organization

The spontaneous emergence of macroscopic nonequilibrium organized structure due to the collective interactions among a large assemblage of simple microscopic objects.

Self-Organized Criticality

Self-organized criticality (SOC) describes a large body of both phenomenological and theoretical work having to do with a particular class of time-scale invariant and spatial-scale invariant phenomena. Fundamentally, SOC embodies the idea that dynamical systems with many degrees of freedom naturally self-organize into a critical state in which the same events that brought that critical state into being can occur in all sizes, with the sizes being distributed according to a power-law. The kinds of structures SOC seeks to describe the underlying mechanisms for look like equilibrium systems near critical points (see Criticality) but are not near equilibrium; instead, they continue interacting with their environment, "tuning themselves" to a point at which critical-like behavior appears. Introduced in 1988, SOC is arguably the only existing holistic mathematical theory of self-organization in complex systems, describing the behavior of many real systems in physics, biology and economics. It is also a universal theory in that it predicts that the global properties of complex systems are independent of the microscopic details of their structure, and is therefore consistent with the "the whole is greater than the sum of its parts" approach to complex systems. Put in the simplest possible terms, SOC asserts that complexity is criticality. That is to say, that SOC is nature's way of driving everything towards a state of maximum complexity.

Simulated Annealing

A mathematical technique for general combinatorial optimization problems. The name comes from the physical process of annealing, during which a material is first heated and then slowly cooled. During annealing, the component atoms of a material are allowed to settle into a lower energy state so that a more stable arrangement of atoms is maintained throughout the cooling process.

Solitons

A mathematically appealing model of real particles is that of solitons. It is known that in a dispersive medium, a general wave form changes its shape as it moves. In a nonlinear system, however, shape-preserving solitary waves exist.

Spatio-Temporal Chaos

A large class of spatially extended systems undergoes a sequence of transitions leading to dynamical regimes displaying chaos in both space and time. In the same way as temporal chaos is characterized by the coexistence of a large number of interacting time scales, spatio-temporal chaos is characterized by having a large number of interacting space scales. Examples of systems leading to spatio-temporal chaos include the Navier-Stokes Equations and reaction-diffusion equations. Coupled-map Lattices have been used for study.

Spin Glasses

A magnetic material whose magnetic magnets respond to both ferromagnetic and antiferromagnetic interactions causing frustration, so that not all the constraints necessary to minimize the system's overall energy can be simultaneously satisfied. There are exponentially stable states, but finding the global ground state is an NP-Hard optimization problem.

Strange Attractors

Describes a form of long-term behavior in dissipative dynamical systems. A strange attractor is an Attractor that displays sensitivity to initial conditions. That it to say, an attractor such that initially close points become exponentially separated in time. This has the important consequence that while the behavior for each initial point may be accurately followed for short times, prediction of long time behavior of trajectories lying on strange attractors becomes effectively impossible. Strange attractors also frequently exhibit a self-similar or fractal structure.

Symbolic Dynamics

Symbolic dynamics is a tool that is used to obtain a coarse-grained representation of dynamical orbits consisting of discrete-symbol sequences. This is done by first partitioning the Phase Space into a finite number of cells C1, C2,…, CN and and focusing on the successive cell-to-cell transitions of the trajectory. The states of the cells, S(C1), S(C2),…, S(CN), are treated as symbols of an N-letter alphabet. Looked at in this way, the continuous dynamics thus induces on the partition a symbolic dynamics describing how the letters of the alphabet evolve in time.

Synergetics

Synergetics refers to what can loosely be called the "European" (vice US) approach to the study of complex systems. Consider a complex system (that is, a system composed of many individual parts) that is controlled from the outside in some manner by a control parameter (say, the system is driven by a constant influx of energy and/or matter). As the control parameter is changed, the system's state can become unstable and be replaced by a new state characterized by particular kinds of spatial, temporal or functional structures. Synergetics consists of strategies of describing what happens when the macroscopic state of systems undergoes a qualitative change. More colloquially, "synergy" is used to refer to how the action of two or more entities ("parts") can achieve an effect that cannot be achieved by any of the parts alone (see Emergence).

Topological Dimension

The topological dimension of object X is an integer defining the number of coordinates needed to specify a given point of X. A single point therefore a topological dimension equal to zero; a curve has dimension one, a surface has dimension two, and so on.

Universality

Universal behavior, when used to describe the behavior of a dynamic system, refers to behavior that is independent of the details of the system's dynamics. It is a term borrowed from thermodynamics. According to thermodynamics and statistical mechanics the critical exponents describing the divergence of certain physical measurables (such as specific heat, magnetization, or correlation length) are universal at a phase transition in that they are essentially independent of the physical substance undergoing the phase transition and depend only on a few fundamental parameters (such as the dimension of the space).

Universal Computer

Universal Turing Machine

Unstable Equilibrium

A stationary state of a dynamical system such that an arbitrarily small perturbation can cause a disturbance of arbitrarily large magnitude. Example: an egg poised on the vertex of a cone.

Hit Counter

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